MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2015/2016

ETN3096 - DIGITAL SIGNAL PROCESSING

(All sections/groups)

1st JUNE 2016
9:00 a.m. – 11:00 a.m.
(2 hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of 8 pages (including cover page) with 4 questions and a list of formulas.

2. Attempt **ALL** questions. This paper carries a total mark of 60. The distribution of the marks for each question is given.

3. Please write all your answers in the answer booklet provided.

Question 1

Figure Q1 shows a signal that continues to negative infinity.

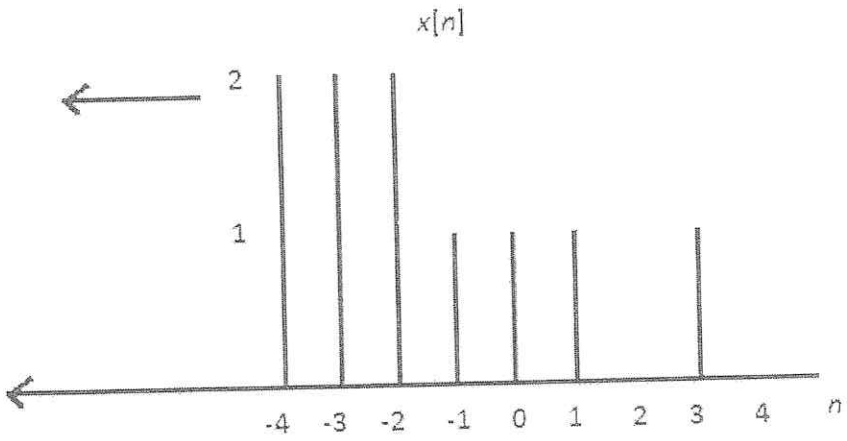

Figure 1

(a) Determine the major characteristics of the signal. Identify at least 5 and state your assumptions.

[5 marks]

(b) Determine the signal representation, in terms of the unit impulse response function or the unit step function, or both.

[3 marks]

(c) Compute the output for the following:

$$y[n] = x[n].u[n] * \{1,1,0,1\}$$

where the * denotes the convolution operation.

[4 marks]

(d) State 3 advantages of Digital Signal Processing.

[3 marks]

Question 2

For each of the following systems, compute the system function and state the region of convergence (ROC):

(a) $h[n] = 2\delta[n] + 3\delta[n-1] + 4\delta[n-2] + \delta[n-4]$

[2 marks]

(b) $h[n] = 4\left(-\frac{1}{3}\right)^{n} u[n] - 3\left(\frac{1}{2}\right)^{n-1} u[-n-2]$

[3 marks]

For the following system function, answer questions (c) to (d):

$$H(z) = \frac{1 - z^{-1}}{(1 - 0.5z^{-1})(1 - 0.25z^{-1})}$$

(c) Obtain the linear constant coefficient difference equation.

[3 marks]

(d) Derive the impulse response of the system assuming the system is realizable.

[4 marks]

(e) Draw the region of convergence, assuming again a realizable system.

[3 marks]

Question 3

(a) Spectral analysis looks at the frequency components of a signal. Explain the differences between a high density spectrum and a high resolution spectrum.

[2 marks]

(b) In digital filter design, Gibbs phenomenon occurs when truncating the ideal filter with a rectangular window. Describe the two problems associated with the Gibbs phenomenon with a diagram and discuss how the two problems can be addressed.

[4 marks]

(c) A Finite Impulse Response (FIR) low-pass filter that can attenuate noise by at least 30 dB without affecting the useful content by more than 3 dB is to be designed. List down all possible windows that can be used in the filter design.

[3 marks]

(d) For all the windows that you listed in 3(c), determine the filter order if the passband and stopband edge frequencies are 0.3π and 0.5π respectively. Suggest which window should be used to design the filter, and justify your selection.

[5 marks]

(e) Give the impulse response, $h[n]$, of the filter in 3(d).

[4 marks]

(f) State two advantages of using an Infinite Impulse Response (IIR) filter instead.

[2 marks]

Question 4

Given the transfer function of a finite impulse response (FIR) filter below:

$$H(z) = (1 - 0.6z^{-1}),$$

$$= 1 - 3.6z^{-1} + 5.4z^{-2} - 4.3z^{-3} + 1.9z^{-4} - 0.5z^{-5} + 0.05$$

(a) Realize it in the following forms:

(i) Direct form;

[3 marks]

(ii) Cascade of three second-order sections.

[3 marks]

(b) Compare the computational complexity of each of the above realizations in terms of multipliers and adders.

[2 marks]

(c) Derive the system function for the structure shown in Figure 4.

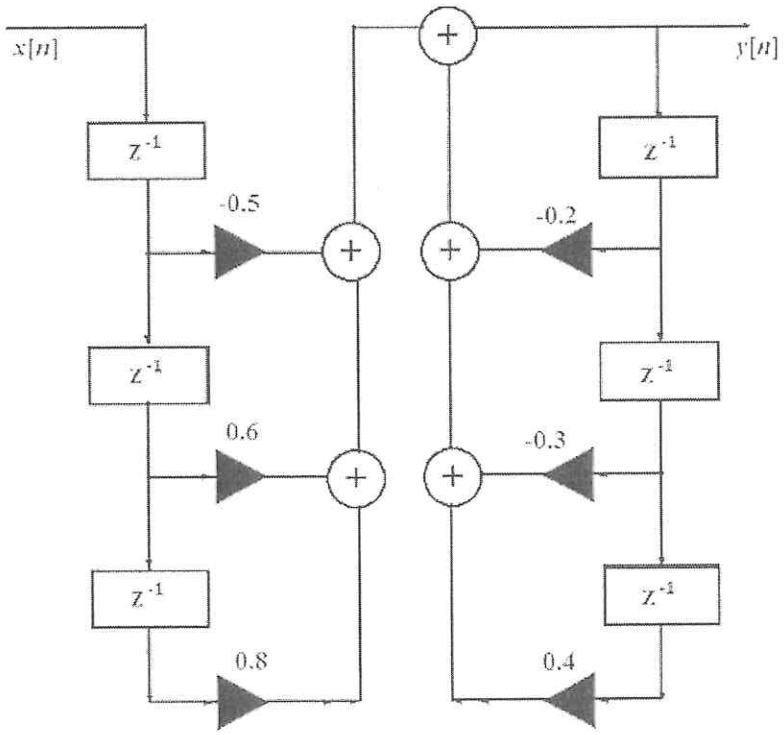

Figure 4.

[2 marks]

End of questions

Appendix

SUMMARY OF USEFUL TABLES

SOME COMMON z-TRANSFORM PAIRS

Sequence	Transform	R.O.C.
1. $\delta[n]$	1	All z
2. $u[n]$	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
3. $-u[-n-1]$	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
4. $\delta[n-m]$	z^{-m}	All z except 0 (if $m>0$) or ∞ (if $m<0$)
5. $a^n u[n]$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
6. $-a^n u[-n-1]$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
7. $na^n u[n]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
8. $-na^n u[-n-1]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
9. $[\cos\omega_0 n]u[n]$	$\dfrac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
10. $[\sin\omega_0 n]u[n]$	$\dfrac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
11. $[r^n\cos\omega_0 n]u[n]$	$\dfrac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
12. $[r^n\sin\omega_0 n]u[n]$	$\dfrac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
13. $\begin{cases} a^n & 0\le n\le N-1 \\ 0, & \text{otherwise}\end{cases}$	$\dfrac{1-a^Nz^{-N}}{1-az^{-1}}$	$\lvert z\rvert > 0$

SOME z-TRANSFORM PROPERTIES

Sequence	Transform	R.O.C.
1. $x[n]$	$X(z)$	R_X
2. $x_1[n]$	$X_1(z)$	R_{X_1}
3. $x_2[n]$	$X_2(z)$	R_{X_2}
4. $ax_1[n]+bx_2[n]$	$aX_1(z)+bX_2(z)$	Contains $R_{X_1} \cap R_{X2}$
5. $x[n-n_0]$	$z^{-n_0}X(z)$	R_X, except for the possible addition or deletion of the origin or ∞
6. $\alpha^n x[n]$	$X(z/\alpha)$	$\lvert\alpha\rvert R_X$
7. $nx[n]$	$-z\dfrac{dX(z)}{dz}$	R_X, except for the possible addition or deletion of the origin or ∞
8. $x^*[n]$	$X^*(z^*)$	R_X
9. $\Re e\{x[n]\}$	$\dfrac{1}{2}[X(z)+X^*(z^*)]$	Contains R_X
10. $\Im m\{x[n]\}$	$\dfrac{1}{2j}[X(z)-X^*(z^*)]$	Contains R_X
11. $x^*[-n]$	$X^*(1/z^*)$	$1/R_X$
12. $x_1[n]*x_2[n]$	$X_1(z)X_2(z)$	Contains $R_{X_1} \cap R_{X2}$
13. Initial-value theorem $x[n]=0,\ n<0$	$\lim_{z\to\infty} X(z)=x[0]$	

CLASSIFICATION OF LINEAR-PHASE FIR SYSTEMS

	$h[n]$ symmetric: $h[n]=h[N-n]$	$h[n]$ antisymmetric: $h[n]=-h[N-n]$
N even	**Type I Linear Phase Filter** $H(e^{j\omega})=e^{-jN\omega/2}\sum_{k=0}^{N/2}a[k]\cos(k\omega)$ $a[0]=h[N/2]$ $a[k]=2h[(N/2)-k]$	**Type III Linear Phase Filter** $H(e^{j\omega})=je^{-jN\omega/2}\sum_{k=1}^{N/2}c[k]\sin(k\omega)$ $c[k]=2h[(N/2)-k]$
N odd	**Type II Linear Phase Filter** $H(e^{j\omega})=e^{-jN\omega/2}\sum_{k=1}^{(N+1)/2}b[k]\cos((k-1/2)\omega)$ $b[k]=2h[\dfrac{(N+1)}{2}-k]$	**Type IV Linear Phase Filter** $H(e^{j\omega})=je^{-jN\omega/2}\sum_{k=1}^{(N+1)/2}d[k]\sin((k-1/2)\omega)$ $d[k]=2h[\dfrac{(N+1)}{2}-k]$

SOME COMMON WINDOWS

Rectangular	$w[n]=\begin{cases}1 & 0\le n\le N\\ 0 & else\end{cases}$
Hanning	$w[n]=\begin{cases}0.5-0.5\cos\left(\frac{2n\pi}{N}\right) & 0\le n\le N\\ 0 & else\end{cases}$
Hamming	$w[n]=\begin{cases}0.54-0.46\cos\left(\frac{2n\pi}{N}\right) & 0\le n\le N\\ 0 & else\end{cases}$
Blackman	$w[n]=\begin{cases}0.42-0.5\cos\left(\frac{2n\pi}{N}\right)+0.08\cos\left(\frac{4n\pi}{N}\right) & 0\le n\le N\\ 0 & else\end{cases}$
Kaiser	$w[n]=\frac{I_0\left(\beta\sqrt{1-\left(\frac{n-\alpha}{\alpha}\right)^2}\right)}{I_0\beta}\quad 0\le n\le N\ \alpha=N/2$ where $\beta=\begin{cases}0.1102(\alpha_s-8.7) & \alpha_s>50\\ 0.5842(\alpha_s-21)^{0.4}+0.07886(\alpha_s-21) & 21\le\alpha_s\le 50\\ 0.0 & \alpha_s<21\end{cases}$ $N=\begin{cases}\frac{\alpha_s-7.95}{14.36\Delta f} & \alpha_s\ge 21\\ 0.9/\Delta f & \alpha_s<21\end{cases}$

COMPARISON OF COMMONLY USED WINDOWS

Type of Window	Peak Side-Lobe Amplitude (Relative)	Approximate Width of Main Lobe	Stopband Attenuation, $20\log_{10}\delta$ (*dB*)	Transition Width (Δf)
Rectangular	-13	$4\pi/(M+1)$	-21	$0.9/M$
Bartlett	-25	$8\pi/M$	-25	
Hanning	-31	$8\pi/M$	-44	$3.1/M$
Hamming	-41	$8\pi/M$	-53	$3.3/M$
Blackman	-57	$12\pi/M$	-74	$5.5/M$

THE COEFFICIENTS IN THE SYSTEM FUNCTION OF A NORMALIZED BUTTERWORTH FILTER (Ω_c=1) FOR ORDERS $1 \le N \le 8$

N	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
1	1.0000							
2	1.4142	1.0000						
3	2.0000	2.0000	1.0000					
4	2.6131	3.4142	2.6131	1.0000				
5	3.2361	5.2361	5.2361	3.2361	1.0000			
6	3.8637	7.4641	9.1416	7.4641	3.8637	1.0000		
7	4.4940	10.0978	14.5918	14.5918	10.0978	4.4940	1.0000	
8	5.1258	13.1371	21.8462	25.6884	21.8462	13.1372	5.1258	1.0000

CLOSED-FORM EXPRESSION FOR SOME USEFUL SERIES

$$\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$$

$$\sum_{n=0}^{N-1} na^n = \frac{(N-1)a^{N+1} - Na^N + a}{(1-a)^2}$$

$$\sum_{n=0}^{N-1} n = \frac{1}{2}N(N-1)$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad |a| < 1$$

$$\sum_{n=0}^{\infty} na^n = \frac{a}{(1-a)^2} \quad |a| < 1$$

$$\sum_{n=0}^{N-1} n^2 = \frac{1}{6}N(N-1)(2N-1)$$

$$\sum_{n=N_1}^{N_2} a^n = \frac{a^{N_1-1} - a^{N_2}}{1-a}$$